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Models with inverse sfermion mass hierarchy and decoupling of the SUSY FCNC effects

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Abstract: We study the decoupling of the first two squark and slepton families in order to lower the flavour changing neutral current effects. Models with inverse sfermion mass hierarchy based upon gauged U(1) flavour symmetries provide a natural framework where decoupling can be implemented. Decoupling requires a large gap between the Fermi scale and the supersymmetry breaking scale. Maintaining the electroweak symmetry breaking at the Fermi scale requires some fine-tuning that we investigate by solving the two-loop renormalization group equations. We show that the two-loop effects are governed by the anomaly compensated by the Green-Schwarz mechanism and can be determined from the quark and lepton masses. The electroweak breaking constraints lead to a small μ scenario where the LSP is Higgsino-like.

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1. Introduction

Many sfermion mass patterns have been suggested in order to alleviate the effects of the flavour non-diagonal contributions in the supersymmetric Flavour Changing Neutral Current (FCNC) problems. Three main mechanisms have been proposed: (i) *degeneracy* in the scalar masses; (ii) *alignment* between fermion and sfermion mass matrices; (iii) *decoupling* of all virtual supersymmetric effects by large scalar masses. Degeneracy seems natural in the context of gravity mediated sypersymmetry breaking although it could also be pointed out that a generic flavour dependence of the Kähler potential tends to spoil this degeneracy in the presence of a spontaneously broken flavour symmetry, if unprotected by this symmetry. In the framework of gauged flavour symmetries the induced D-type soft masses are especially dangerous in that respect. Actually, gauge mediation of supersymmetry breaking at a relatively low scale would lead to negligible FCNC effects if the gravitino is light enough but it apparently faces inherent problems to induce the correct electroweak symmetry breaking.

The implementation of the alignment prescription via a rather ad hoc choice of flavour symmetries has been advocated [1] to be the only natural solution, although some of the arguments used have to be revised in the framework of broken supergravity theories. The examples of such symmetries are sufficiently contrived that one feels compelled to look for another possibility. On the other hand, decoupling has always served as a possible remedy for the inconsistencies of any model beyond the standard theory. However, third generation scalars together with the Higgs and gaugino sectors which control the electroweak symmetry breaking should be kept light enough to avoid too much fine-tuning. Since FCNC effects are more stringent within the first two generations of quarks and leptons, it has been envisaged that the first two generations could be considerably heavier than the other supersymmetric particles [2] — actually, an efficient suppression of FCNC effects by decoupling alone would require very heavy masses. As a matter of fact, the first and second families of sfermion masses do enter the MSSM expression for the Z boson mass when one takes into account the two loop effects. This has been used to put limits on the mass gap between generations of scalars, mostly by requiring the absence of an excessive fine-tuning. Other bounds have been obtained from the positivity of the stop masses [3]. We will comment on these generic bounds later on.

Realizations of supersymmetric models with such a pattern of scalar masses, coined "inverse hierarchy models", have been proposed in different frameworks. It has been shown that models with an anomalous U(1) explaining the fermion mass hierarchies [4, 5], naturally lead to an inversion of the sfermion spectrum with respect to the corresponding fermion masses ordering [6, 7]. Other models provide an inverse hierarchy whose origin springs from extra gauge symmetries at the unification scale [8, 9].

Recently a new mechanism based on the existence of a quasi-fixed point in the renormalization group evolution of scalar masses has been used in order to generate dynamically a hierarchy between the first two families and the third one [10]. These models are based upon the possibility for families with large Yukawa couplings to evolve faster than families with smaller Yukawa couplings. On the one hand this mechanism provides a dynamical explanation to the existence of an inverse hierarchy. On the other hand the initial conditions have to be tailored in order to be in the domain of attraction of the fixed points.

Our main interest here is to cast the decoupling approach within spontaneously broken supersymmetry scenarios. In this context a very large gap between families seems to be difficult to reach in the supergravity mediated framework due to the

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form of the scalar masses, assumed to be flavour diagonal, for simplicity,

$$m_i^2 = m_{3/2}^2 + \frac{\langle F_a \rangle \langle \bar{F}_{\bar{b}} \rangle}{M_P^2} \partial_a \partial_{\bar{b}} \ln(\partial_i \partial_{\bar{i}} K) + X_i \langle D_X \rangle \,, \tag{1.1}$$

where K is the Kähler potential, X_i is the gauge charge of the state whilst $\langle F_a \rangle$ and $\langle D_X \rangle$ are the auxiliary fields responsible for supersymmetry breaking, which (vectorially) add up to $\sqrt{3} m_{3/2} M_P$. This formula displays the dependence of the sfermion masses on: (i) flavour independent terms such as $m_{3/2}^2$; (ii) $\langle F_a \rangle$ along directions where there is a dependence of the Kähler metric in the matter field families; (iii) $\langle D_X \rangle$ along flavour symmetries. One easily realizes from (1.1) that scalars in different generations cannot be split by more than one order of magnitude, unless (i) is compensated by (ii), like in no-scale supergravity.

In the presence of supersymmetry breaking and flavour symmetry breaking there are two sources of "induced" supersymmetry breaking capable to yield flavour dependent scalar mass splittings. Let us consider a simple situation [4] in order to illustrate this point: an abelian flavour symmetry $\mathrm{U}(1)_X$ broken by the value of a Frogatt-Nielsen field ϕ with $m_{3/2} \leq \langle \phi \rangle \leq M_P$. Assuming a vanishing cosmological constant, from the supergravity lagrangians one obtains [6, 7, 11] the induced supersymmetry breaking $\langle F_\phi \rangle \sim m_{3/2} \langle \phi \rangle$ along the ϕ direction and $\langle D_X \rangle \sim m_\phi^2$ where $m_{3/2}^2$ is the total supersymmetry breaking and m_ϕ^2 is the ϕ soft mass, of $O(m_{3/2}^2)$. In this case, the D-type splitting is always relevant, the F-type one being proportional to $\langle \phi \rangle^2 / M_P^2$. This flavour dependence in the soft scalar masses can have several physical consequences that are investigated in this paper.

1.1 iMSSM

We concentrate here on a class of inverse hierarchy models models which we denote iMSSM. They are based on the assumption that an anomalous $\mathrm{U}(1)_X$ gauge symmetry is present in the flavour sector of the theory. The anomaly is fixed by the Green-Schwarz mechanism which then determines the scale of the flavour symmetry breaking $\langle \phi \rangle$. The induced D-term produces a mass splitting, $m_i^2 - m_j^2 = (X_i - X_j) m_\phi^2$. In models where the anomalous $\mathrm{U}(1)_X$ is responsible for the fermion mass hierarchy the charge difference are roughly related to the fermions masses [6]

$$m_i^2 - m_j^2 \propto \ln \left(\frac{m_j^F}{m_i^F} \right) . \tag{1.2}$$

This leads to an inverse hierarchy in the sfermion masses compared to the fermionic hierarchy (quarks and leptons).

This fact was first pointed out in the framework of general broken supergravity coupled to abelian flavour gauge symmetry [6] and, subsequently, in models with dynamical supersymmetry breaking [7]. It has been noticed that, because the top

Yukawa coupling is of $O(1)$, there is the relation $(X_{t_L}+X_{t_R}+X_{H_2})=0$, for the charges of the top-Higgs sector, implying that the soft mass combination, $(m^2_{U_3}+m^2_{Q_3}+m^2_{H_2})$ receives no contribution from the $D-$terms. Since this is the combination appearing in the one-loop correction to the boson mass M_Z^2, the radiative gauge symmetry breaking is automatically protected, at one-loop, against large $D-$terms that are due to *any* gauge symmetry broken at scales below M_P.

It goes without saying that the FCNC effects are particularly dangerous in the iMSSM framework. It has been suggested that this problem can be evaded by a suitable combination of degeneracy, alignment and, last but not least, decoupling of the first two generations [6, 12]. The latter has prompted us to evaluate the two-loop corrections. This is done in section 2.

1.2 A limit on two-loop effects

In section 3, we show that for inverse hierarchy models based on D-term splitting the dominant contribution to the two loop terms in the renormalization evolution of the scalar masses is proportional to the anomalies of the $\mathrm{U}(1)_X$ group.[1]

For the most relevant case of gauged $\mathrm{U}(1)_X$ symmetries the anomalies must be cancelled by the Green-Schwarz mechanism. We will concentrate on theories such as the weakly coupled heterotic string with only one anomalous $\mathrm{U}(1)_X$ and one dilaton-axion to implement the Green-Schwarz mechanism.

An interesting aspect of this result is that despite the large variety of X charges and choice of symmetry to explain the fermion hierarchy the $\mathrm{U}(1)_X$ anomaly can be fixed in a rather model independent way. Indeed by using the previously obtained relations between the anomaly $\mathcal{A}$ and the fermion masses one gets [5]

$$m_u m_c m_t (m_e m_\mu m_\tau)^3 (m_d m_s m_b)^{-2} \approx \left(\frac{g_X^2 \mathcal{A}}{32\pi^2}\right)^{\mathcal{A}/2} \sin^3\beta \cos^3\beta (174\,\mathrm{GeV})^6 \,, \qquad (1.3)$$

where the quark and lepton masses are taken at the scale $g_X\sqrt{\mathcal{A}}\,M_P/4\pi$, $\tan\beta$ is the ratio between the two Higgs vacuum expectation values (vev's), g_X is the $\mathrm{U}(1)_X$ gauge coupling and $\mathcal{A}$ is the $\mathrm{U}(1)_X$ anomaly with respect to the standard model gauge groups. The evaluation of (1.3) yields $\mathcal{A} \approx 25 \pm 3$. Remarquably enough this leads to a prediction of the Cabibbo angle $\theta_C \approx 0.2$ in these models. This establishes a quite model independent estimate of the two loop effects in inverse hierarchy models. It turns out to be much smaller than values considered in previous discussions inspired by these models [2, 3].

[1] This conclusion has already been reached in [9] for models with a D-term scalar mass. However the approach there is different from ours. The anomalous $\mathrm{U}(1)_X$ charges are chosen to shift the first and second generation scalar masses while keeping a light sector for the third generation. The anomaly is then compensated by the introduction of heavy extra-matter to stabilize the spectrum. Our analysis includes the effects of the anomaly due to the MSSM states, which is directly related to the fermion masses as below.

1.3 Maximal hierarchy limit

As already stressed,in the iMSSM context, the supersymmetric flavour problem requires very heavy sfermions in the first two generations, hence a large supersymmetry breaking scale, $m_{3/2}^2 \gg M_Z^2$. The fine-tuning problem becomes crucial. In the cMSSM, where the coefficient of the universal soft scalar masses m_0^2 in the expression for M_Z^2 is strongly suppressed — a Nature fine-tuning of the top mass m_t — for moderate values of $\tan\beta$, the necessary fine-tuning is mostly between $M_{1/2}^2$ and μ^2. This requires relatively large values for μ, yielding gaugino-like LSP. Several studies in the literature [13] have already discussed how the predictions are modified by departing from the cMSSM scalar degeneracy. The main point is that, in this case, the scalar masses can also participate in cancelling the $M_{1/2}^2$ term in the expression for M_Z^2, allowing for relatively low values of μ. For these small μ solutions, the spectrum will consist of scalars heavier than gauginos, which are also heavier than the higgsinos. The advantage of this class of models is that the fine-tuning now occurs mostly in the ratio $M_{1/2}^2/m_{3/2}^2$, which is more obviously related to the supersymmetry breaking mechanism, while the origin of the μ parameter, which could now be of $O(M_Z)$, remains more mysterious.

In section 4, we investigate the possibility of large values of $m_{3/2}^2$ together with small μ values. The two-loop correction, controlled by the anomaly, contributes in some cases. In this regime, the iMSSM does reveal an inverted mass spectrum as compared with the cMSSM. We discuss approximate constraints in the neighbourhood of the 'infinitely' fine-tuned solution for large $\tan\beta$, but this rough approximation turns out to be quite appropriate from our numerical analysis.

The iMSSM mass patterns are displayed in section 5. We summarize our conclusions in the last section. The case of more than one U(1) flavour symmetries is sketched in the appendix.

2. Two loop renormalization effects

Let us first treat the two loop renormalization group equations [14] of the scalar masses in the following approximation,

$$\frac{dm_i^2}{dt} = -8\pi^2 C_A(i)\left[M_A^2 - \frac{g_A^2}{16\pi^2}\,\mathrm{tr}\left(\frac{C_A m^2}{d_A}\right)\right] + 2g_1^2 Y_i\left[S + \frac{g_A^2}{4\pi^2}\,\mathrm{tr}(Y C_A m^2)\right], \quad (2.1)$$

where the indices $A = 1, 2, 3$ correspond to the gauge group factors U(1), SU(2), SU(3) respectively, g_A is the corresponding gauge coupling, d_A the algebra dimensions, M_A is the gaugino mass and $C_A(i)$ is the Casimir eigenvalue for the fermion/sfermion labelled by i. Finally,

$$S = \mathrm{tr}(Y m^2) \quad (2.2)$$

introduces a Y-dependent term in the scalar masses.

The approximation (2.1) is appropriate for the class of models discussed herein and corresponds to neglecting the two-loop corrections proportional to M_A^2 which are suppressed by a factor of $g_A^2 C_A(i)/16\pi^2$ with respect to the one loop terms. As displayed, (2.1) is not valid for the stops. Including the top Yukawa couplings to extend the calculation to the third family scalar masses and consistently neglecting the terms in M_A^2 which are higher order in g_A^2, yields the solution

$$\begin{aligned} m_i^2 = m_i^2(0) - \frac{a(i)}{12}\rho\left(3\bar{m}^2 + 8M_{1/2}^2 + (1-\rho)(A_{Q_3}(0) + 2M_{1/2})^2 - \delta\right) + \\ + |t| C_A^2(i) g_A^2 \left(8\left(g_A^2 + \frac{1}{2}\right) M_{1/2}^2 - \delta\right) + \frac{Y_i}{22}[S(t) - S(0)] \end{aligned} \tag{2.3}$$

at the Fermi scale where the coefficients $a(i)$ are $a(U_3) = 4$, $a(Q_3) = 2$, $a(H_2) = 6$ and zero otherwise, $3\bar{m}^2 = (m_{U3}^2 + m_{Q3}^2 + m_{H2}^2)$, A_{Q_3} is the soft coupling associated to the top Yukawa coupling, and finally,

$$\rho = \frac{h_t}{(h_t)_{\text{F.P.}}} \approx \frac{0.72}{\sin^2\beta}\,. \tag{2.4}$$

In approximating the solutions for m_i^2 we are anticipating and taking advantage of the fact that the two-loop term

$$\delta = \frac{1}{4\pi^2}\,\mathrm{tr}\left(\frac{C_A m^2}{d_A}\right)\Bigg|_{t=0} \tag{2.5}$$

is almost independent of the index A in the classes of models considered here, that we now turn to discuss.

2.1 In the cMSSM

The most important effect of the radiative corrections on the $\mathrm{SU}(2)\times\mathrm{U}(1)$ breaking appears in the Higgs parameter

$$\begin{aligned} m_{H_2}^2 \approx m_{H_2}^2(0) + 0.52(M_{1/2}^2 - 0.15\delta) - 0.014 S_0 - \\ - \frac{0.36}{\sin^2\beta}\left(3\bar{m}^2 + 8M_{1/2}^2 + \left(1 - \frac{0.72}{\sin^2\beta}\right)(A_0 + 2M_{1/2})^2 - \delta\right)\,. \end{aligned} \tag{2.6}$$

Therefore the two loop effects due to possible heavy scalars in the first two generations become relevant for $\delta \sim O(\text{few} M_{1/2}^2)$. For instance assuming a degeneracy amongst the heavy scalars of mass m in the first two generations this corresponds to $m \sim 5M_{1/2}$.

It is well known that in the cMSSM with boundary conditions $m_i^2(t=0) = m_0^2$ the coefficient of m_0^2 in M_Z^2 is small for large values of $\tan\beta$. Indeed the dependance of $m_{H_2}^2$ on m_0^2 in the $\tan\beta \gg 1$ limit is

$$m_{H_2}^2 \approx -0.1 m_0^2 + 0.3\delta - 2.76 M_{1/2}^2\,. \tag{2.7}$$

Taking the traces over all the MSSM scalars one gets $\mathrm{tr}(C_3/8) = 6$, $\mathrm{tr}(C_2/3) = 7$, $\mathrm{tr}\, C_1 = 6.6$, so that

$$\delta \approx \frac{m_0^2}{2\pi} \,. \tag{2.8}$$

The possibility of obtaining a relatively large value of m_0^2 without worsening the fine-tuning in M_Z^2 remains at the two-loop level.

3. Anomalous U(1) models

As already emphasized in the introduction the natural realization of the inverse hierarchy for squarks and sleptons occurs in models where the fermion mass hierarchy is explained by a Frogatt-Nielsen mechanism with an anomalous $\mathrm{U}(1)_X$ local flavour symmetry. Recently it has been advocated that in type-IIB orientifold models of string theory one could expect different anomalous $\mathrm{U}(1)_X$ with their anomalies being cancelled by a corresponding number of moduli fields [15]. We shall ignore this possibility and remain within the more traditional heterotic-like picture with only one anomalous $\mathrm{U}(1)_X$ [16]. Of course one could postulate the existence of other anomaly-free U(1) flavour gauge symmetries in order to explain the fermion mass hierarchy. As discussed in the appendix the results are essentially similar in the multi-U(1) models.

We refer to the comprehensive literature on this subject for the details and quote the main results only. The anomaly cancellation is provided by the Green-Schwarz mechanism. A necessary condition for the compensation of the anomalies with respect to the standard model gauge symmetries as well as the gravitational anomaly is the equality

$$\mathcal{A}_1 = \mathcal{A}_2 = \mathcal{A}_3 = \frac{\mathrm{tr}(X)}{24} \,, \tag{3.1}$$

where $\mathcal{A}_A = 2\,\mathrm{tr}(XC_A)/d_A$. The Kac-Moody levels k_A have all been taken to be one to simplify the discussion. The coefficiient of the Fayet-Iliopoulos term required by the $\mathrm{U}(1)_X$ gauge invariance — which is nothing but the contribution of the dilaton to D_X — is given by

$$\xi^2 = \frac{k_X g_X^2 \mathcal{A}}{32\pi^2} \,. \tag{3.2}$$

Introducing a Froggat-Nielsen field ϕ which is standard model gauge singlet with a charge $X = -1$ (by a suitable normalization of the $\mathrm{U}(1)_X$ charges), the $\mathrm{U}(1)_X$ gauge symmetry is broken at the minimum of the scalar potential where

$$\begin{aligned} |\phi|^2 &= \xi^2 M_P^2 \,, \\ \langle F_\phi \rangle &\sim m_{3/2} \xi M_P \,, \\ \langle D_X \rangle &= m_\phi^2 \,, \end{aligned} \tag{3.3}$$

where m_ϕ^2 is the soft mass given by the supersymmetry breaking mechanism. The $\langle F_\phi \rangle$ and $\langle D_X \rangle$ vev's are the induced supersymmetry breaking terms. The former contributes to the scalar masses only as $\xi^2 m_{3/2}^2$ and is neglected here.[2] The latter gives an important D-term contribution to the scalar masses so that at the scale ξM_P,

$$m_i^2 = \hat{m}_i^2 + X_i \langle D_X \rangle \ , \tag{3.4}$$

where $\hat{m}_i^2$ is the contribution from the supersymmetry breaking independent of the D_X breaking and the charges X_i are model dependent.

3.1 Two-loop correction and anomaly

In the inverse hierarchy models the two loop contributions of the heavy scalars to the renormalization group equations turns out to be quite independent of the choice of charges. Indeed, one gets from the definition (2.5) and the masses (3.4),

$$4\pi^2 \delta = \sum_i \frac{C_A(i)}{d_A} (\hat{m}_i^2 + X_i \langle D_X \rangle) \, , \tag{3.5}$$

which yields

$$\delta = \hat{\delta} + \frac{1}{8\pi^2} \mathcal{A} \langle D_X \rangle \ . \tag{3.6}$$

In obtaining this result we have used the equality of the anomalies (3.1). Hence, the main two-loop contribution, coming from the D-terms in the scalar masses, is proportional to the anomaly $\mathcal{A}$. Of course, the two-loop corrections coming from D-terms corresponding to non-anomalous U(1)$'$s cancel.

Interestingly enough, the anomaly $\mathcal{A}$ can be calculated from its relation to the fermion masses. Indeed, even if the X-charges that control the fermion masses are model dependent, it is possible to display a combination of masses that only depends on the charges through $\mathcal{A}$, as we now turn to discuss.

3.2 Calculation of the anomaly

In the anomalous U$(1)_X$ approach to the fermion hierarchy, with the abelian flavour symmetry breaking given by the small parameter ξ as discussed above, the quark and lepton Yukawa couplings to the Higgses are given by

$$Y_{f_i} \sim \xi^{f_{Li} + f_{Ri} + h} \, , \tag{3.7}$$

where the fermion name (e.g., q_i) also denotes its X-charge (resp., $X(u_{Li}) = X(d_{Li})$), and h is the X-charge of the appropriate Higgs field. In particular, one obtains, from the values of the third generation Yukawa couplings, the relations:

$$\begin{aligned} &q_3 + u_3 + h_2 \approx 0 \, , \\ &q_3 + d_3 + h_1 \approx l_3 + e_3 + h_1 \approx \frac{4 - \ln \tan\beta}{\ln \xi} \, , \end{aligned} \tag{3.8}$$

[2] Though, as discussed in [6], this contribution is relevant in the discussion of FCNC effects.

and ξ is roughly the Cabibbo angle, $\theta_C \approx \xi$. The charges of the other family fermions are more or less fixed by their Yukawa couplings and the Kobayashi-Maskawa mixings.

Because of the condition (3.1) on the anomalies, $\mathcal{A}$ is related to the fermion masses as follows,

$$m_u m_c m_t \left(m_e m_\mu m_\tau\right)^3 \left(m_d m_s m_b\right)^{-2} \approx \xi^{\mathcal{A}} \sin^3\beta \cos^3\beta (174\,\mathrm{GeV})^6 \,, \tag{3.9}$$

where $\xi^2 = g_X^2 \mathcal{A}/32\pi^2$, while for the Higgs charges one obtains,

$$(h_1 + h_2) \ln \xi \approx \ln \left(\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \right) . \tag{3.10}$$

The quark and lepton masses are defined at the scale ξM_P. Putting the experimental masses in (3.9) yields

$$\mathcal{A} \ln \left(\frac{32\pi^2}{g_X^2 \mathcal{A}} \right) \approx (90 \pm 3 \ln \sin 2\beta) \,, \tag{3.11}$$

giving $\mathcal{A} \approx 25 \pm 3$ and $\xi \approx 0.2$, with the GUT value $g_X^2 = .5$. This is in reasonable agreement with the relation $\theta_C \approx \xi$. From (3.10) one gets $(h_1 + h_2) \approx 0$, a result that we shall use later. In the appendix we discuss the model dependence of these relations.

Therefore, as a typical result, the relevant two-loop contribution to the low-energy scalar masses (2.3) is given by a relatively low value,

$$\delta \approx \frac{m_\phi^2}{\pi} + \hat{\delta} \,, \tag{3.12}$$

where we have used (3.3).

3.3 i+cMSSM

Let us first evaluate the impact of this two loop correction in a simple model (i+cMSSM) with universality assumed for the primordial supersymmetry breaking, namely, a contribution m_0^2 to all scalar soft masses, and with an anomalous $\mathrm{U}(1)_X$ flavour symmetry as discussed above. In this case, $\langle D_X \rangle = m_0^2$, and the scalar masses at the flavour symmetry breaking scale are

$$m_i^2 = m_0^2 \left(1 + X_i\right) . \tag{3.13}$$

From (3.8), the parameter $\bar{m}^2$ of the one-loop correction in (2.3) is equal to m_0^2, and, from (3.12) and (2.8), the two-loop contribution depends on

$$\delta \approx m_0^2 \left(\frac{1}{2\pi} + \frac{1}{\pi} \right) \approx 0.5 m_0^2 \,. \tag{3.14}$$

The corresponding contribution to $m^2_{H_2}$ is

$$\Delta m^2_{H_2} \approx \frac{0.2 m_0^2}{\sin^2\beta}\,, \tag{3.15}$$

which is about three times larger than the corresponding parameter in the cMSSM. As we shall discuss in the next section, contrarily to what happens in the cMSSM, this alone could be enough to allow for small μ models, in the large $\tan\beta$ limit, even if $X(H_2) = 0$.

3.4 iMSSM

We now turn to discuss a more elaborate model [6] exhibiting the inverse hierarchy for the scalar masses, where some flavour dependence is incorporated into the Kähler potential besides the anomalous $\mathrm{U}(1)_X$ gauged flavour symmetry and the Green-Schwarz mechanism. The key assumption, which allows the model to be predictive, is that the *fermion mass hierarchies* are fixed solely by the abelian flavour symmetries, not by the moduli dependence. Amazingly, this assumption turns out to imply that the $(\text{mass})^2$ differences between sfermions with the same SM quantum numbers are proportional to the gravitino $(\text{mass})^2$, $m^2_{3/2}$. Even without specifying the primordial supersymmetry breaking along the dilaton and moduli directions in the iMSSM, the sfermion masses can be parameterized in a simple and suggestive way, which we now turn to summarize.

Let us denote by Φ_i the matter superfields and their scalars where $\Phi = Q$, U, D, L, E refers to the standard model fields and $i = 1, 2, 3$ refers to the family index. We denote by ϕ_i the X charges. At the scale ξM_P the soft terms satisfy the relations

$$\begin{aligned}
m^2_{\Phi_i} - m^2_{\Phi_j} &= (\phi_i - \phi_j) m^2_{3/2} \\
m^2_{U_3} + m^2_{Q_3} + m^2_{H_2} &= M^2_{1/2} \\
m^2_{D_3} + m^2_{Q_3} + m^2_{H_1} &= M^2_{1/2} + (d_3 + q_3 + h_1) m^2_{3/2} \\
m^2_{E_3} + m^2_{L_3} + m^2_{H_1} &= M^2_{1/2} + (e_3 + l_3 + h_1) m^2_{3/2} \\
m^2_{H_2} + m^2_{H_1} &= (2 + h_2 + h_1) m^2_{3/2} \\
A_{U_i} &= (u_i + q_i + h_2) m_{3/2} - M_{1/2} \\
A_{D_i} &= (d_i + q_i + h_1) m_{3/2} - M_{1/2} \\
A_{L_i} &= (e_i + l_i + h_1) m_{3/2} - M_{1/2} \\
B &= (2 + (h_2 + h_1)\theta(h_2 + h_1))\, m_{3/2}\,.
\end{aligned} \tag{3.16}$$

Actually, the terms proportional to the $\mathrm{U}(1)_X$ charges come out as a particular combination of the $\langle D_X \rangle$ induced breaking and the supersymmetry breaking in the moduli sector, which give rise to this general form for the mass splitting between families. We have only considered the soft terms which are diagonal in the family indices, although the pattern of the off-diagonal terms give constraints on the $\mathrm{U}(1)_X$ charges from the FCNC bound. Other relations for the soft terms will be spelt out later.

The fermion hierarchy requires a relatively strong family ordering of the charges $\phi_1 > \phi_2 > \phi_3$. We concentrate on this situation and even more on the case where $M_{1/2} < m_{3/2}$. However we do not have to impose any particular choice for the charges ϕ_i, in many of the physical issues discussed below since the two loop corrections that are relevant to the inverse hierarchy scenario are controlled by the anomaly. This is fixed by the fermion masses to

$$\delta = \frac{\mathcal{A} m_{3/2}^2}{8\pi^2} \approx \frac{m_{3/2}^2}{\pi} \,. \tag{3.17}$$

With the relations in (3.16) one has for instance

$$m_{H2}^2 = m_{H2}^2(t=0) - \frac{0.36}{\sin^2\beta}\left(\left(10 - \frac{0.72}{\sin^2\beta}\right) M_{1/2}^2 - \delta\right) + 0.52(M_{1/2} - 0.15\delta)\,. \tag{3.18}$$

Therefore, the two-loop corrections are basically negligible in the iMSSM. Nevertheless, the δ term is of some importance in the discussion of the next section.

4. Higgsino-like LSP and electro-weak symmetry breaking

It is well-known that by departing from the universality assumptions of the cMSSM many of its striking predictions are dramatically affected. In particular the nature of the LSP can change. This is mainly a matter of competition between the μ^2 and $M_{1/2}^2$ parameters that appear with different signs in the supersymmetric expression for M_Z^2. The cMSSM coefficient of the universal soft scalar masses m_0^2 is strongly suppressed for rather large values of $\tan\beta$. As a general rule, the necessary fine-tuning between $M_{1/2}^2$ and μ^2 then favours a lighter gaugino than the Higgsino.[3] In this section we show that in the iMSSM discussed in the previous sections, the Higgsino turns out to be a natural option for the LSP. Let us sketch the situation within an analytic approximation to the supersymmetric $\mathrm{SU}(2) \times \mathrm{U}(1)$ breaking. In terms of $t = \tan\beta$, the minimum equations read

$$\begin{aligned} m_{H_1}^2 - m_{H_2}^2 - B\mu t\left(1 - \frac{1}{t^2}\right) &= M_Z^2 \frac{1-t^2}{1+t^2} \\ m_{H_1}^2 + m_{H2}^2 + 2\mu^2 - B\mu t\left(1 + \frac{1}{t^2}\right) &= 0 \end{aligned} \tag{4.1}$$

at the classical level. The radiative corrections are important but the main contributions can be included by redefining

$$\begin{aligned} \hat{m}_1{}^2 &= m_{H_1}^2 - \frac{M_Z^2}{2}\frac{1-t^2}{1+t^2} + 3\frac{h_t^2}{16\pi^2}\mu^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) \\ \hat{m}_2{}^2 &= m_{H_2}^2 + \frac{M_Z^2}{2}\frac{1-t^2}{1+t^2} + 3\frac{h_t^2}{16\pi^2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2) \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right). \end{aligned} \tag{4.2}$$

[3] This applies mostly to the case $M_{1/2} \sim O(m_0)$. If m_0 is much larger than the other parameters the pattern may change. A recent analysis of these patterns can be found in ref. [17], where they are nicely summarized in a figure.

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From the minimum equations we deduce that

$$\mu = \frac{Bt}{2}\left(1 \pm \sqrt{1-\frac{4\hat{m}_1^2}{B^2t^2}}\right), \tag{4.3}$$

so that the small μ solution leading to a Higgsino-like LSP corresponds to the minus sign in the previous equation. For large enough values of t (to be discussed later) such that $0 < 4\hat{m}_1^2 \ll B^2t^2$, one gets the following relations

$$\begin{aligned} \mu &\approx \frac{\hat{m}_1^2}{Bt}\left(1+\frac{\hat{m}_1^2}{B^2t^2}\right) \\ \hat{m}_2^2 &\approx \frac{\hat{m}_1^2}{t^2}\left(1-\frac{\hat{m}_1^2}{B^2}\right). \end{aligned} \tag{4.4}$$

Namely the small μ regime corresponds to $\mu^2 \propto t^{-2}$ and $m_{H_2}^2 \propto t^{-2}$. This suggests to expand the solutions in powers of t^{-2}. This is quite unphysical but mathematically sound. Let us start with an approximation to the low energy masses parameterized as follows

$$\begin{aligned} m_{H_1}^2 &= m_0^2(1-\sigma_H) + \frac{1}{2}(M_{1/2}^2 - 0.15\delta) \\ m_{H_2}^2 &= m_{H1}^2 + 2\sigma_H m_0^2 - 0.36\left(1+\frac{1}{t^2}\right)\left(3\bar{m}^2 + 8M_{1/2}^2 + \Delta - \delta\right), \end{aligned} \tag{4.5}$$

where

$$\Delta = \left(0.28 - \frac{0.72}{t^2}\right)\left(A_{U_3} + 2M_{1/2}\right)^2 \tag{4.6}$$

is model dependent. In (4.5), m_0 is the universal scalar mass in the cMSSM and $m_0 = m_{3/2}$ in the iMSSM, as discussed before, and we have assumed $m_{H_1}^2 + m_{H_2}^2 = 2m_0^2$ as consistent with $h_1 + h_2 = 0$.

As a first approximation we determine $M_{1/2}^2/m_0^2$ by taking the large fine-tuning limit, $M_Z^2 \ll m_0^2$. We also neglect the radiative corrections and we keep only the relevant powers of $\tan\beta$. Then, one can solve (4.4) for $M_{1/2}$ in each of the models discussed in the previous sections. For the sake of illustration, we take the values predicted by the iMSSM, $A_{U_3} = -M_{1/2}$ and $B = 2m_0$, but the latter only enters into the term $\propto t^{-2}$.

(*a*) *cMSSM.* In this model, $\sigma_H = 0$, $\bar{m}^2 = m_0^2$, $\delta \approx m_0^2/(2\pi)$. One gets

$$\frac{M_{1/2}^2}{m_0^2} \approx -0.02 - \frac{0.7}{t^2}, \tag{4.7}$$

which excludes mathematically the small μ solution in the limit $m_0^2 \gg M_Z^2$. However, by putting back the radiative corrections (i.e., top-stop loops), the r.h.s. of (4.7) changes sign for t large enough. Actually, in the final expression for $m^2_{H_2}$, the result of the cancellation between the tree level and the MSSSM one-loop running of the term proportional to m_0, the top-stop correction in (4.5) and the two-loop contributions turn out to have roughly the same order of magnitude. Of course, this entails that a precise determination of the ratio (4.7) would require a complete two loop calculation [17].

(*b*) *i+cMSSM.* In this case, the universality is only broken by the $\mathrm{U}(1)_X$ D-terms, so that $\bar{m}^2 = m_0^2$ from (3.8), $\delta \approx 3m_0^2/(2\pi)$ from (3.14), $\sigma_H = h_2$, yielding,

$$\frac{M_{1/2}}{m_0^2} \approx 0.04 + 0.4\sigma_H - \frac{0.7 + 0.3\sigma_H}{t^2}\,. \tag{4.8}$$

The existence of this solution, especially for $h_2 = 0$, is due to the larger two-loop contributions. The gauginos are much lighter than the sfermions for these small μ solutions. In this example, a cancellation must occur between the one-loop term in $M^2_{1/2}$ and the two-loop term in m_0^2 for large values of the soft masses.

(*c*) *iMSSM.* It is characterized by $\bar{m}^2 = M^2_{1/2}$, from (3.16), and $\delta \approx m^2_{3/2}/\pi$, from (3.17). This leads to

$$\frac{M^2_{1/2}}{m^2_{3/2}} \approx 0.36(1 + \sigma_H) + .05 - \frac{0.7 + 0.3\sigma_H}{t^2}\,. \tag{4.9}$$

Roughly, the parameter σ_H can take values in the range $[-1, 0]$. E.g., if we take the value $\sigma_H = -0.25$ and $t = 2$, we get 0.17 for the ratio (4.9), which is close to the values obtained in a scanning of the parameter space. Notice that the two-loop (anomaly) term contributes by about one-third to this result. The ratio in (4.9) means a real fine-tuning, and in our numerical analysis (after reintroduction of the radiative corrections and M_Z in the expressions) the deviations from this 'infinite fine-tuning' limit are rather small. In order to allow for a big hierarchy in the sfermion masses, one has to take rather small values of $M_{1/2}/m_{3/2}$, by increasing $|\sigma_H|$. This ratio is related to the Goldstino angle $\sin\theta_G = M_{1/2}/\sqrt{3}m_{3/2}$. In a sense this is a better variable to be tuned than $\mu/m_{3/2}$ since it is simply related to the nature of the supersymmetry breaking. Still it has to be fine-tuned to match a quantity which, in the iMSSM, depends on the parameter σ_H, related to the properties of the Higgs fields under the $\mathrm{U}(1)_X$ and modular symmetries.

Notice that the condition for a Higgsino-like LSP, $\mu^2 < M^2_{1/2}/6$, is fulfilled with $\tan\beta > 3$, for $\sigma_H > -.75$. Otherwise, the LSP can be gaugino-like, while the lightest chargino remains Higgsino-like. Therefore, the small μ solution of the iMSSM is generically characterized by (i) large values of $m_{3/2}$, i.e., very heavy sfermions of the two first generations; (ii) smaller values of $M_{1/2}$, i.e., moderate gaugino masses; and (iii) μ as low as $O(M_Z)$, i.e., a Higgsino as the LSP. Radiative corrections have been neglected in this discussion, but the main effect of their inclusion, in this case, is to increase the value of $\tan\beta$ for a given set of parameters.

5. The iMass spectrum

In this section we shall discuss the typical mass spectrum that one can derive from inverse hierarchy models. We will also comment briefly on the predictions for the FCNC effects, a main issue in these models because of the large mass splitting between the families.

The mass spectrum of the iMSSM version [6] depends on three parameters σ_H, σ_L and σ_Q which measure the departure from scalar mass universality between the two Higgs doublets, the leptons and the quarks of the third family, respectively. These parameters define the solutions of (3.16) and so include the dependence on the corresponding X-charges and, in this model, on their flavour dependent Kähler geometry. They are family independent. Then, the family dependent mass terms, accordingly to (3.16), depend only on the X-charge differences, e.g., $q_1 - q_3$. Such charges can be chosen to get a good agreement with fermionic mass patterns and mixing angles. The choice of charges plays also a role in the S term in the masses (at the Fermi scale). The correction to the masses due to this term is

$$\delta m_i^2 = \frac{Y_i}{22}(S - S_0)\,, \tag{5.1}$$

where S_0 comprises a term like $\mathrm{tr}(XY)m_{3/2}^2$. From the renormalization group equations one finds that the evolution of S is given in first approximation by

$$S - S_0 = \left(\frac{g_1^2}{g_0^2} - 1\right) S_0 \approx -0.6 S_0\,. \tag{5.2}$$

The effect of this contribution has been often overestimated in the literature. In any instance, the term (5.1) can be consistently included in the definition of σ_H, σ_L and σ_Q, without loss of generality. This is understood in what follows.

The masses of the third family sleptons are then given by ($\kappa = M_Z^2 \cos 2\beta$)

$$\begin{aligned}
m_{\tilde\tau_L}^2 &= (1+\sigma_l)m_{3/2}^2 + 0.5(M_{1/2}^2 - 0.15\delta) + 0.4\kappa \\
m_{\tilde\nu_L}^2 &= (1+\sigma_l)m_{3/2}^2 + 0.5(M_{1/2}^2 - 0.15\delta) + -0.5\kappa \\
m_{\tilde\tau_R}^2 &= 1.16M_{1/2}^2 - (\sigma_l - \sigma_H)m_{3/2}^2 - 0.03\delta + 0.23\kappa
\end{aligned} \tag{5.3}$$

and the third family squark masses are given by

$$\begin{aligned}
m^2_{\tilde b_L} &= (1+\sigma_q)m^2_{3/2} + 6.9M^2_{1/2} - 0.5\delta - \frac{\rho}{6}((10-\rho)M^2_{1/2} - \delta) + 0.42\kappa \\
m^2_{\tilde b_R} &= -(\sigma_q - \sigma_H)m^2_{3/2} + 7.4M^2_{1/2} - 0.43\delta + 0.75\kappa \\
m^2_{\tilde t_L} &= (1+\sigma_q)m^2_{3/2} + 6.9M^2_{1/2} - 0.5\delta - \frac{\rho}{6}((10-\rho)M^2_{1/2} - \delta) - 0.35\kappa \\
m^2_{\tilde t_R} &= -(2+\sigma_q+\sigma_H)m^2_{3/2} + 7.4M^2_{1/2} - 0.44\delta - \frac{\rho}{3}((10-\rho)M^2_{1/2} - \delta) - 0.15\kappa\,.
\end{aligned} \tag{5.4}$$

Since we are interested in the case $M_{1/2} \ll m_{3/2}$, the allowed range for the parameters σ_H, σ_L and σ_Q, is strongly constrained, and σ_H has also to be consistent with the electroweak break conditions (4.4).

Let us now present some generic features of the spectrum. The differences in the charges between the first two generations and the third one are model dependent to some extent. However, one can minimize these uncertainties by considering the combinations that are more directly related to the fermion masses, as given by (3.7). For the first family sfermions, as compared to the third family ones, one finds,

$$\begin{aligned}
m^2_{\tilde e_L} + m^2_{\tilde e_R} - m^2_{\tilde \tau_L} - m^2_{\tilde \tau_R} &\approx \frac{\ln(m_e/m_\tau)}{\ln\xi} m^2_{3/2} \\
m^2_{\tilde d_L} + m^2_{\tilde d_R} - m^2_{\tilde b_L} - m^2_{\tilde b_R} &\approx \frac{\ln(m_d/m_b)}{\ln\xi} m^2_{3/2} \\
m^2_{\tilde u_L} + m^2_{\tilde u_R} - m^2_{\tilde t_L} - m^2_{\tilde t_R} &\approx \frac{\ln(m_u/m_t)}{\ln\xi} m^2_{3/2}\,.
\end{aligned} \tag{5.5}$$

These are high energy relations that remain valid at low energies as far as the masses of the third generation are taken from (5.3) and (5.4) without the terms proportional to ρ. Analogous expressions hold for the second generation of sfermions. Of course, parity conjugated sfermions are usually splitted by a large amount with respect to the above averages. If we take, as an example, $\sigma_H \approx \sigma_L \approx \sigma_Q \approx -1$, which leads to relatively light charginos (without further motivation for this particular choice), all the third family sfermions are as light as the gauginos, the first and second families are much heavier. The two-loop contributions are important in this case where the fine-tuning between $M_{1/2}$ and $m_{3/2}$ is large.

In the numerical analysis, the radiative corrections are included, and the parameter space is scanned around the maximal fine-tuning values. As expected we find Higgsino-like LSP's degenerate with the lightest chargino. The chargino masses can be as low as 100 GeV. We have explicitly cut the spectrum by (arbitrarily) imposing that the MSSM Higgs mass is greater than 100 GeV. We do find Higgses within the $100 - 109$ GeV slot corresponding to a value of $\tan\beta$ which does not exceed 18. Among the squarks the left sbottoms are the lightest. We present in figure 1 the mass spectrum as a function of σ_H. We have rescaled the masses and display them in units of $m_{3/2}$. As expected the hierarchy between families is not destroyed by the

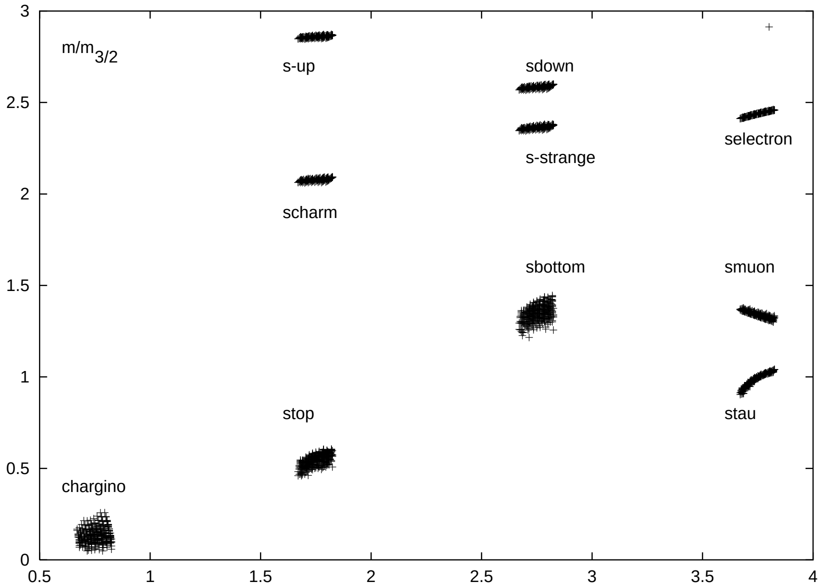

Figure 1: The mass spectrum in units of $m_{3/2}$. We have displayed the lightest chargino mass and the left-right average masses for the U squarks, the D squarks and the sleptons as a function of σ_H appropriately shifted.

evolution down to the Fermi scale. The values of $m_{3/2}$ chosen in the figure are below 2 TeV. Higher values of $m_{3/2}$ would not modify the picture, only the fine-tuning would be more severe.

Let us come back to one of the issues which prompted our study: the FCNC effects and the decoupling of the first two families. The mass insertions that are usually used to evaluate the FCNC contributions [18] are roughly given in terms of the X charges of the particles by

$$\delta_{ij} \sim 2\frac{|X_i - X_j|}{X_i + X_j}\xi^{|X_i - X_j|}\,. \tag{5.6}$$

The strong constraints on the mass insertions with $i = 1$ and $j = 2$, suggests [6] a choice of some degeneracy and some alignment in the diagonal soft masses by choosing $d_1 = d_2$ and $e_1 = e_2$. However, this is not enough and we still need large values of the supersymmetry breaking parameter, $m_{3/2} \sim 2\,\text{TeV}$ for a sufficient FCNC decoupling. Indeed, as noticed before, the flavour dependence of the soft terms coming from the $\langle F \rangle$ supersymmetry breaking have also to be taken into account. They are reduced by at least a factor ξ^2, as follows from (3.3), and more model dependent, but still dangerous enough for the $K - \bar{K}$ system. Fortunately we do get such high values of $m_{3/2}$ in our numerical scanning without further effort. Yet, the contribution to ϵ_K comes out close to the phenomenological bounds in this model, in spite of the combined use of all three anti-FCNC mechanisms, degeneracy and alignment from the equality of some charges, together with decoupling through a relatively large supersymmetry breaking scale in the scalar sector.

Let us now comment on the possible implications for the B physics. In the insertion approximation, the parameters δ_{23} and δ_{13}, for both chiralities, can be obtained from the charge differences, $d_1 - d_3$, $d_2 - d_3$, $u_1 - u_3$, $u_2 - u_3$, $q_1 - q_3$, $q_2 - q_3$, which are all positive in these models. Using charge differences that are consistent with fermion masses and mixings in (5.6), the δ_{13} and δ_{23} turn out to be all below the present limits from B physics [19]. Since one stop and the (higgsino-like) chargino can be relatively light, the insertion approximation may be a poor one. If one takes into account the numerical analysis of limits on the MSSM parameters from B physics, as given, e.g., in ref. [19, 20], one sees that the present model has several characteristics that tend to reduce the FCNC effects in the B system: the chargino and the lighter stop are lighter than the charged Higgs, which is predicted to be quite heavy, and the quark-squark misalignement for the third family is characterized by small angles. This implies that our model is generically consistent with the present B physics data, for large m_0. Still, a detailed analysis would certainly give some restrictions on the parameter space of the model. And the presently running and future B experiments could even challenge this class of models. However, the limits on CP violating soft terms are less compelling as their phases are arbitrary parameters in these models. Of course, a more elaborated flavour theory — e.g., with an additional U(1) — could lower the FCNC effects [6].

It is worth noticing that the small $M_{1/2}/m_{3/2}$ and $A_t/m_{3/2}$ ratios that characterize these models are what is needed [21] to avoid charge and colour breaking vacua without need for further cosmological assumptions. In the large μ version of the model, one can reduce the hierarchy and increase the degeneracy by increasing the $M_{1/2}/m_{3/2}$ ratio. Besides the fact that it would bring back the issue of a fine-tuning of the $\mu/M_{1/2}$ ratio, this would be strongly constrained by the wrong vacuum problems.

6. Summary and concluding remarks

We have studied the decoupling of the first two squark families in order to lower the FCNC effects. This has been done using the gauged U(1) flavour symmetries which had already been utilised to explain the fermion masses and the mixing angles. Within this framework we have focused on more model-independent results. In particular as soon as one tries to induce large mass hierarchies one faces the fine tuning problem in the electroweak sector. Indeed the Fermi scale has to be maintained although the supersymmetry breaking scale is pushed up beyond the TeV limit. This forces to study carefully the diverse compensations in the M_Z^2 equation. As a result one has to resort to a two-loop analysis of the Higgs sector. Fortunately we have shown that the two-loop effects are solely governed by the Green-Schwarz anomaly which is determined from the fermion masses. This allows a thorough study of the minimum equations, and the possibility of a scenario where the fine-tuning appears in the $(M_{1/2}, m_{3/2})$ sector with a small value for μ. This differs from the

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usual cMSSM where μ is large. This leads to a Higgsino-like LSP and a characteristic mass spectrum. In particular we find light charginos. On the contrary, the sfermions of the first and second families should be of order a few TeV, a nice experimental signature, indeed.

Of course, the inverse hierarchy models based on abelian flavour symmetries are especially affected by the FCNC problems. The supersymmetry breaking scale required to get an efficient decoupling would be very high. Therefore, it is not clear whether they are a good choice to escape the flavour changing effects in spite of the natural prediction of heavy sfermions in the first two families. On the other hand they also possess some other nice features: a natural small scale from the Fayet-Iliopoulos term, the presence of anomalous abelian symmetries in superstring solutions, the simplicity of the fitting to the puzzling fermion hierarchy and, last but not least, the relation between the fermion and sfermion spectrum. A compromise could be obtained with additional flavour symmetries, for instance non-abelian ones, to reduce the splitting between the sfermion in the first two families. More speculatively, one could hope that the more recent developments in string theory – see for instance [15, 22] and references therein – would provide new insights into the old quarrel of supersymmetry with flavour.

A. Anomalies and multiple U(1)'s

In the case of more than one abelian flavour charges, X_i, $(i = 1, \ldots, n)$, we introduce an equal number of scalars, Φ^i, so that all the U(1)'s are broken. For the consistency of the model, we make the following assumptions:

A) Only one U(1) symmetry is anomalous, which we call X_1, and only the corresponding D-term has a Fayet-Iliopoulos term with coefficient ξ. This is related to the anomaly $\mathcal{A}$ through the Green-Schwarz mechanism by (3.2). It is mandatory that the other abelian charges fulfil the analogous of (3.1) and that they do not introduce any other anomaly.

B) Let us denote by $-\phi_{ij}$ the charge X_i of the scalar Φ^j. They are chosen so that there is no term in the superpotential with the Φ's alone. These U(1) charges are normalized so that all the corresponding coupling constants are equal.

The relevant soft-terms are the masses m_i of the scalars Φ^i. Let ϕ^{-1}_{ij} be the inverse of the charge matrix ϕ defined above, which has an inverse because of our assumption B). The equivalent of (3.3) is now

$$\begin{aligned} |\phi_i|^2 &= \phi^{-1}_{i1}\xi^2 M_P^2\,, \\ \langle D_{X_i}\rangle &= m_j^2 \phi^{-1}_{ji}\,. \end{aligned} \tag{A.1}$$

Then, (3.9) is modified by the replacement,

$$\xi^{\mathcal{A}} \longrightarrow \prod \left(\phi^{-1}_{i1}\xi^2\right)^{\mathcal{A}_i/2} , \tag{A.2}$$

where $\mathcal{A}_i = \phi^{-1}_{i1}\mathcal{A}$. Therefore, in the pluri-U(1) case, the resulting value for the anomaly can be slightly different from the value in section 3.

Finally, the contributions to the sfermion masses from the $\langle D_{X_i}\rangle$ terms are $m_j^2\phi^{-1}_{ji}X_i(a)$, where $X_i(a)$ is the corresponding fermion charge. The contribution to the two-loop scalar masses becomes,

$$\delta = \hat{\delta} + \frac{1}{8\pi^2}m_j^2\phi^{-1}_{j1}\mathcal{A}\,, \tag{A.3}$$

where δ is defined in (2.5). This allows for some variation with respect to the values discussed in section 3, but the two-loop contributions generically remain as small.

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